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## Superconformal Invariant Two- and Three-Point Functions and Invariant Equations in Two-Dimensional Space-Time

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(Summary)

The representations of the superconformal Lie algebra ((2,1) and (2,2)) are found (2,6). The invariant two-point function has the form

$$F(x_1, x_2, \theta_1, \theta_2) = N_{\{d,\nu\}} \exp\{-i\bar{\theta}_1 \widehat{\partial}\theta_2\} \left(\frac{x_{12}^2}{4}\right)^{-d} \left(\frac{x_-^{12}}{a_+^{12}}\right)^{\nu},$$

where $N_{\{d,\nu\}}$ is a normalization constant, d is the scale dimension of the supermultiplet and $\nu$ labels the irreducible representation of the same multiplet with respect to the $SO$ (1,1) Lorentz subgroup. The three-point function is given by (3,17) and (3,18).

The invariant action is given in the from (4,1), (4,3), (4,7) and (4,9). From this action, the Euler-Lagrange equations have been derived (see (4,4), (4,8), (4,11), (4,13) and (4,18)). A class of the classical solutions of these equations is given by (4,19).

# On the Class of Two-Dimensional Scalar Supersymmetric Models with Higher Quantum Conservation Laws

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It is shown, at least for weak coupling, that the supersymmetric Sine-Gordon model is the only one in the class of two-dimensional scalar supersymmetric models with non-derivative interactions, having an infinite set of local quantum conservation laws.

Recently, supersymmetric extensions of several most interesting two-dimensional completely integrable models (supersymmetric Sine-Gordon (SSG) [1], O($N$) nonlinear sigma- [2], $CP^{N-1}$ chiral field- [3]) were proposed. In particular, for the SSG model an infinite series of classical local conservation laws was constructed [4], as well as the exact quantum S-matrix (without kinks) was found [5].

It is well known that the existence of infinite sets of local conservation laws implies nontrivial restrictions on the dynamics (absence of multiparticle production and factorization [6] of scattering processes) and thus it is the main ingredient in the exact solution of the corresponding quantum models [7]. However, due to nonmultiplicative renormalizations of composite operators, quantum equations of motion (QEM) do not in general resemble the analogous clasaical ones and, consequently, the powerful formalism of the inverse scattering method in classical, completely integrabl , models cannot directly carry over to the quantum theory. Therefore, a proper construction of higher local quantum conserved currents (HLQCC) turns out to be a major step to the exact solvability.

In the present note, within the framework of sypersymmetric normal product formalism, we investigate the class of two-dimensional scalar supersymmetric models with nonderivative interactions in the light of searching for HLQCC. It turns out that, at least in the weak coupling scheme, the SSG model is the only one in the above class, having a first nontrivial HLQCC (besides the usual spin-vector supercurrent). One can further construct the whole infinite series of HLQCC for the SSG model [8], adopting the arguments from ref. [9]. Thus, our result provides a complete dynamical justification of the corresponding exact $S$-matrix found in [5].

In the following we shall use the standard superspace formalism [10]. Supersymmetric covariant derivative $\mathfrak{D}=\partial/\partial\bar{\theta}-i(\gamma^{\mu}\theta)\partial/\partial x^{\mu}$ looks in components like:

$$\mathfrak{D}_1=\partial/\partial\theta^2-i\theta^2\partial/\partial\eta,\quad \mathfrak{D}_2=-\partial/\partial\theta^1-i\theta^1\partial/\partial\xi$$

$$\xi=1/2(x^0+x^1),\quad \eta=1/2(x^0-x^1),$$

where the following representation for the $\gamma$-matrices is chosen

$$\gamma_0=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \gamma_1=\begin{pmatrix}0&1\\-1&0\end{pmatrix};\ \hat{p}\equiv p^{\mu}\gamma_{\mu}=\begin{pmatrix}0&p_0-p_1\\p_0+p_1&0\end{pmatrix}.$$

General supersymmetric models of a single scalar superfield $\Phi(x,\theta)$ with nonderivative selfcouplings are defined by the Lagrangian

$$\mathscr{L}(x,\theta)=1/2\mathfrak{D}_2\Phi\mathfrak{D}_1\Phi+m/_2\Phi^2+\mathscr{L}_I(\Phi). \tag{1}$$

For the SSG model $m/_2\Phi^2+\mathscr{L}_I(\Phi)=-m/\beta^2\cos\beta\Phi$. Supersymmetric perturbation theory [10, 11] is described in terms of vertices determined by $\mathscr{L}_I(\Phi)$ and propagators

$$D(p;\ \theta,\theta')=e^{\bar{\theta}\hat{p}\theta'}(1-m\delta(\theta-\theta'))\,[m^2-p^2-i0]^{-1}. \tag{2}$$

To renormalize supergraphs we employ the supersymmetric extension [12] of the usual BPHZ scheme [13]. In order to ensure minimal subtractions we shall use the trick of partially "soft" mass [14], i. e. we replace $m$ in the numerator of (2) by $sm$, where s $(0\leq s\leq 1)$ is an auxiliary parameter, and define subtraction operators $\tau^{\omega}_{p,s}$ by the properties [12]

$$\tau^{\omega}_{p,s}f(p,s)=t^{\omega}_{p,s}f(p,s);\ \tau^{\omega}_{p,s}\theta^{\alpha}f(p,s;\theta)=\theta^{\alpha}\tau^{\omega+1/2}_{p,s}f(p,s;\theta), \tag{3}$$

$f(p,s;\theta)$ being an arbitrary function and $t^{\omega}_{p,s}$ the standard Taylor operators of order $\omega$ in the variables p, s. After implementing all subtractions in Zimmermann's "forest formula" [13] for an arbitrary supergraph $\Gamma$ we set $s=1$ The corresponding ultraviolet degrees $\omega(\Gamma)$ are defined as follows ($V(\Gamma)$ — number of vertices in $\Gamma$)

$$\omega(\Gamma)=2(1-V(\Gamma)); \quad \omega(\Gamma)=\dim P-2V(\Gamma). \tag{4}$$

The first formula in (4) refers to usual supergraphs, the second one to supergraphs with one extra composite operator P vertex insertion (this is the only case we shall need).

QEM [15] and Zimmermann's identities (ZI) [13] for normal products of composite operators from the basis for systematic algebraic construction of HLQCC in two-dimensional completely integrable models [16, 9]. In the present sypersymmetric context, QEM can be derived by means of the standard BPHZ procedure [15] with the only difference that, due to the partially "soft" mass renormalization (3), one should use a modified identity for the propagator in momentum space

$$[\mathfrak{D}_2\widetilde{\mathfrak{D}}_1-sm]\left\{e^{\bar\theta\,\widehat{p}\,\theta'}\frac{1-sm\delta(\theta-\theta')}{m^2-p^2-i0}\right\}=\delta(\theta-\theta')+m^2(s^2-1)\frac{\delta(\theta-\theta')}{m^2-p^2-i0}. \tag{5}$$

In this way we obtain ($\langle\cdots\rangle$ denote time-ordered Green's functions, $Q(\mathfrak{D})$ arbitrary differential monomial in $\mathfrak{D}_a$):

$$\begin{aligned}\langle\mathfrak{N}[PQ(\mathfrak{D})(\mathfrak{D}_2\mathfrak{D}_1\Phi)](x,\theta)X_\Phi\rangle&=m\langle\mathfrak{N}[PQ(\mathfrak{D})\Phi](x,\theta)X_\Phi\rangle\\&+\langle\mathfrak{N}[PQ(\mathfrak{D})(\delta\mathfrak{L}_I/\delta\Phi)](x,\theta)X_\Phi\rangle-i\sum_{l=1}^{L_\Phi}[Q(\mathfrak{D})\delta(x-x_l)\delta(\theta-\theta_l)]\\&\times\langle\mathfrak{N}[P](x,\theta)\widehat{X}^l_\Phi\rangle-m^2\langle\mathfrak{N}[PQ(\mathfrak{D})\{(s^2-1)\Phi\}](x,\theta)X_\Phi\rangle;\end{aligned} \tag{6}$$

$$X_\Phi\equiv\prod_{l=1}^{L_\Phi}\Phi(x_l,\theta_l),\quad \widehat{X}^l_\Phi\equiv\prod_{l'=1,\,l'\neq l}^{L_\Phi}\Phi(x_{l'}\theta_{l'}).$$

Here the symbol $\mathfrak{N}$ denotes a canonical (minimally subtracted) normal product except in the last anisotropic ("anomalous") term on the r. h. s. of (6). The meaning of the curly brackets is the following. Firstly, the propagator corresponding to $\Phi$ inside the curly brackets is not (2) but a modified one: $\delta(\theta-\theta')[m^2-p^2-i0]^{-1}$ (cf. (5)) (graphically, it will be represented by a marked line, see Fig. 1). Secondly, one assigns to each subgraph $\dot\Gamma$, containing the modified propagator, an oversubtraction degree $\widetilde{\omega}(\dot\Gamma)=\omega(\dot\Gamma)+2=\omega(\Gamma)+1$, where $\Gamma$ is topologically identical to $\dot\Gamma$ but with all propagators normal.

In analogy with the usual nonsupersymmetric case, let us look for a HLQCC which would give rise to the conservation law

$$\sum_{\text{in}}(p_{l,\eta})^3=\sum_{\text{out}}(p_{l',\eta})^3; \quad p_\eta=p_0-p_1, \tag{7}$$

where the sums run over the sets of in- and out-particle momenta for an arbitrary scattering process. Eqn. (7) would imply absence of multiparticle production in two-particle collisions and factorization of the three particle $S$-matrix to products of two-particle ones (i. e. (7) would lead to the exact determination of the two-particle amplitudes [7]). The most general admissible supersymmetric structure for such a HLQCC reads

$$J^1_{7/2}(x,\theta)=\mathfrak{N}[\mathfrak{D}^3_1\Phi\mathfrak{D}^4_1\Phi+g\mathfrak{D}_1\Phi(\mathfrak{D}^2_1\Phi)^3](x,\theta),$$

(8)
$$J^2_{7/2}(x,\theta)=m\mathfrak{N}[A_1(\Phi)\mathfrak{D}^5_1\Phi+A_2(\Phi)\mathfrak{D}_1\Phi\mathfrak{D}^4_1\Phi$$
$$+A_3(\Phi)\mathfrak{D}^2_1\Phi\mathfrak{D}^3_1\Phi+A_4(\Phi)\mathfrak{D}_1\Phi(\mathfrak{D}^2_1\Phi)^2](x,\theta),$$

where $A_i(\Phi), i=1,\ldots,4$ are a priori arbitrary entire functions in $\Phi(x,\theta)$ and g is a constant. They are to be determined by the requirement that the Ward identity for the conservation of $J^a_{7/2}(x,\theta)$ holds

(9)
$$\mathfrak{D}_2\langle J^1_{7/2}(x,\theta)X_\Phi\rangle-\mathfrak{D}_1\langle J^2_{7/2}(x,\theta)X_\Phi\rangle=\text{contact terms}.$$

To this end we substitute (8) into (9) and use QEM (6). The main point here is to evaluate the anisotropic normal products

(10)
$$m^2\langle\mathfrak{N}[\mathfrak{D}^4_1\Phi\mathfrak{D}^2_1\{(s^2-1)\Phi\}+\mathfrak{D}^3_1\Phi\mathfrak{D}^3_1\{(s^2-1)\Phi\}$$
$$+g(\mathfrak{D}^2_1\Phi)^3\{(s^2-1)\Phi\}+3g(\mathfrak{D}^2_1\Phi)^2\mathfrak{D}_1\Phi\mathfrak{D}_1\{(s^2-1)\Phi\}](x,\theta)X_\Phi\rangle$$

in terms of canonical normal products by means of the ZI. Exactly the same reasoning as in the case of the usual SG model [16] shows that the only sub-

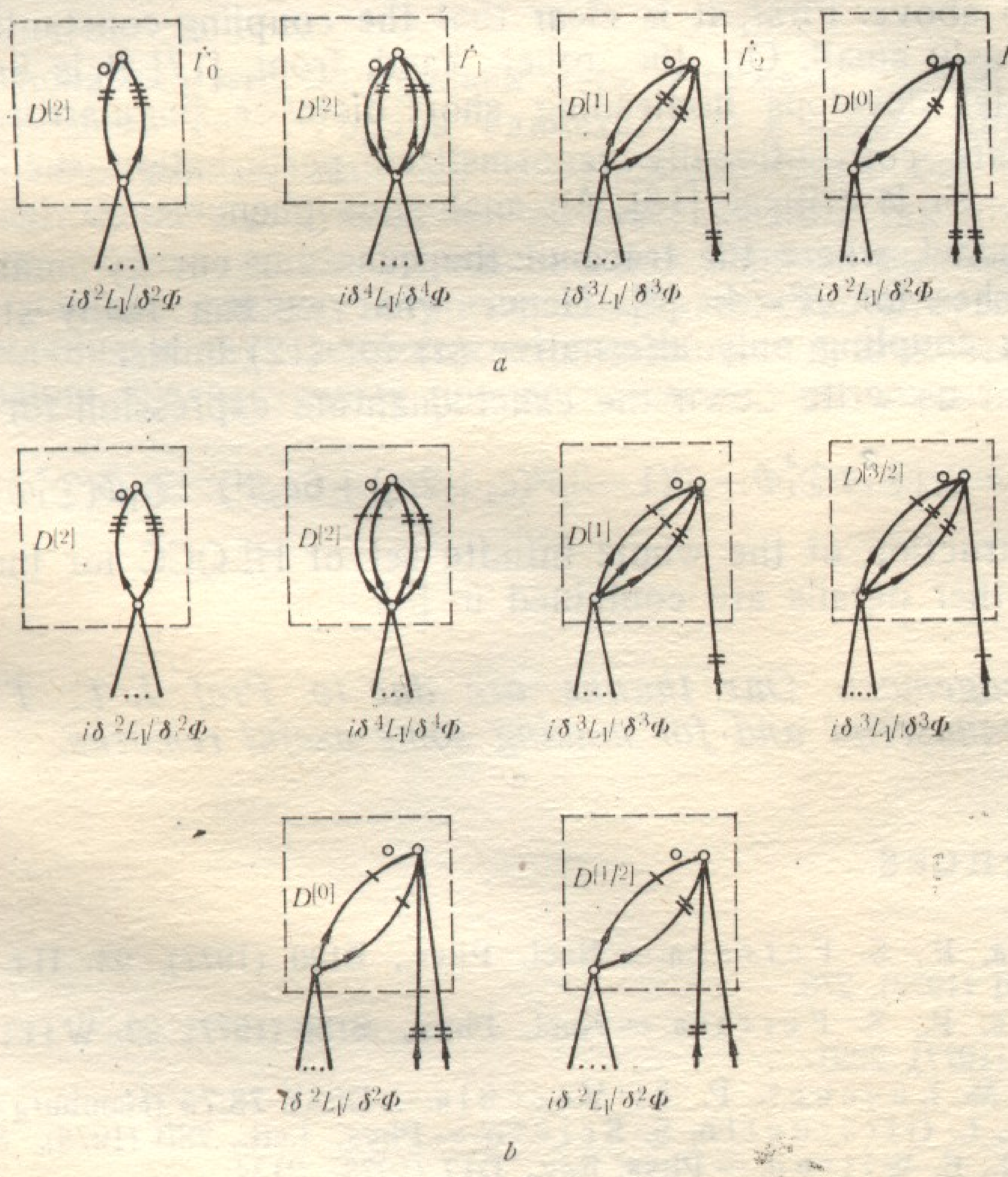

Fig. 1

graphs $\dot\Gamma$ giving rise to the ZI for (10) are those with $V(\dot\Gamma)=1$(see Fig. 1; $D^{[\omega]}\equiv\tau^{\omega}_{p,s}-\tau^{\omega-1}_{p,s}$, bars denote $\mathfrak{D}_1$-derivatives). The contributions of the box subgraphs in Fig. 1$a$ are $c_k/m^2$, $k=0,\ldots,3$ respectively, with $c_k$ — computable constants. The contributions of the remaining ones in Fig. 1$b$ are simply expressed in terms of $c_k$. On account of these facts, the condition that (9) should hold is reduced to a set of 4 linear equations for $A_i(\Phi)$ plus the most important ordinary differential equation for $V(\Phi)\equiv 1/2m\Phi^2+\mathcal{B}_I(\Phi)$

(11) $$6c_2 g\delta^5 V/\delta^5\Phi-[1-3g(c_2+2c_3)]\delta^3 V/\delta\Phi^3+g\delta V/\delta\Phi=0.$$

The formal solution of (11) respecting space-reflection symmetry $\Phi\to-\Phi$ (under which $J^1_{7/2}(x,\theta)$ is already invariant) reads

(12) $$V(\Phi)=-\widetilde{m}_1/\beta_1^2\cos\beta_1\Phi-\widetilde{m}_2/\beta_2^2\cos\beta_2\Phi,$$

$$g=-\beta_j^2[1-3\beta_j^2(c_2+2c_3)+6c_2\beta_j^4],\quad j=1,2,$$

where two alternatives hold: $(a)\widetilde{m}_1\neq 0$, $\beta_1\equiv\beta$ arbitrary, $\widetilde{m}_2=0$ (or vice versa) — SSG model; $(b)$ $\widetilde{m}_j\neq 0$, $j=1,2$, $\beta_1^2\beta_2^2=(6c_2)^{-1}>(4\pi)^2$.

Clearly, we could proceed exactly in the same way on a classical level and would again arrive at eqn. (11) but without terms containing $c_k$. Therefore, the only possible solution in this case is the classical SSG model. Thus, the appearing of larger formal solution (12) is entirely due to quantum effects (i. e. due to the presence of the "anomalous" terms (10)).

However, there are some arguments which make uncertain the validity of alternative $(b)$ above. First, it is clear that the coupling constants $\beta_1$, $\beta_2$ cannot be simultaneously small. On the other hand, from [17] it is known that the usual SG model develops nonleading short distance singularities for $\beta^2_{SG}\geq 4\pi$, which invalidate conventionally renormalized perturbation theory. Moreover, the point $\beta^2_{SG}=8\pi$ is critical [18]. An analogous phenomenon takes place also in the SSG model, where the trace of the quantum energy-momentum tensor formally vanishes for $\beta^2=4\pi$ [8]. Hence what we can really state is that at least for weak coupling only alternative $(a)$ for (12) holds.

Finally, let us write down the exact quantum expression for $J^1_{7/2}$:

$$J^1_{7/2}(x,\theta)=\Re[\mathfrak{D}_1^3\Phi\mathfrak{D}_1^4\Phi-\beta^2(1-3\beta^2(c_2+2c_3)+6c_2\beta^4)^{-1}\mathfrak{D}_1\Phi(\mathfrak{D}_1^2\Phi)^3](x,\theta).$$

The construction of the whole infinite set of HLQCC for the SSG model, as well as further details are contained in [8].

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## О классе двумерных скалярных суперсимметричных моделей с высшими квантовыми законами сохранения

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(Резюме)

В работе показано, что по крайней мере в режиме слабой связи суперсимметричния модель синус-Гордон единственная в классе двумерных скалярных суперсимметричных моделей с взаимодействием без производных, являющаяся вполне интегрируемой как на классическом, так и на квантовом уровне. Найдено точное явное выражение для первого высшего сохраняющегося тока, откуда следует строгое доказательство отсутствия множественного рождения в двухчастичных столкновениях и замечательного свойства факторизации трехчастичных процессов рассеяния, описываемых данной моделью.